QUANTUM PHYSICS I - Oct. 31, 2019

Write your name and student number on **all** sheets. There are four problems in this exam. You can earn 90 points in total.

PROBLEM 1: WAVEFUNCTION and PROBABILITY (5+5+5+5+10=30 points)

Consider a particle in one dimension with a wavefunction $\psi(x,t)$.

a) Explain in one or two sentences what happens to the wavefunction (in the orthodox interpretation) when an observer measures a specific observable.

b) Explain in one or two sentences (no formulae) why the agnostic position (that states that the difference between the realist and the orthodox interpretations boils down to philosophy) has lost most of its support in the second half of the previous century.

c) Prove that, when the potential energy $V(x)$ is symmetric around a point, e.g. $x = 0$, a symmetric wavefunction will continue to be symmetric and similarly for an anti-symmetric wavefunction.

d) Explain in one or two sentences what is meant by the normalisation of the wavefunction of the particle.

e) Using the time-dependent Schrödinger equation, show that the total probability is time-independent.

PROBLEM 2: (NOT SO) FREE PARTICLE (5+5+5+5=20 points)

Consider a particle with a potential energy that vanishes everywhere except between 0 and a, where it takes a constant positive value V_0.

a) First consider the free particle (i.e. take $V_0 = 0$). What are the stationary states and the corresponding eigenvalues of the Hamiltonian? Are these also momentum eigenstates?

b) Now consider a non-vanishing energy treshold V_0, and an incoming wave from the left with an energy E between 0 and V_0. Write down the general form of such a wavefunction, and give the constraints that are imposed by boundary conditions.

c) Calculate the transmission rate (as a function of all parameters, including E, a and V_0) by solving the boundary conditions of the previous question.

d) What would the transmission rate be in the classical limit $\hbar \to 0$?

PROBLEM 3: QM in 2D (5+5+10=20 points)

Consider the wavefunction of a particle in two spatial dimensions with a spherically symmetric potential energy. The Laplacian in 2D polar coordinates is given by

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\,. \tag{1}$$

a) Separation of variables makes an Ansatz for the wavefunction of the form $\psi = f(t)R(r)\Theta(\theta)$. What does the time-independent Schrödinger equation imply for the angular dependence of the wavefunction, i.e. which differential equation does $\Theta(\theta)$ have to satisfy? Write down the most general solution to this differential equation and discuss the boundary conditions.

b) Similar for the radial dependence of the wavefunction: using the redefinition $R(r) = u(r)/\sqrt{r}$, which differential equation does $u(r)$ have to satisfy?

c) Consider a spherically symmetric potential of the form $V(r)$ given by the infinite square well ranging from $r = 0$ to $r = a$. Derive the ground state of this system, imposing the correct boundary conditions. Make sure that it is normalisable but do not bother with the exact normalisation.

PROBLEM 4: SPIN STATES (5+5+10=20 points)

The Pauli matrices represent the spin operators on a two-dimensional spin space, relevant for an $s = 1/2$ particle. Consider instead an $s = 1$ particle, with a three-dimensional spin space.

a) Which two operators can you construct from the spin operator that commute and hence are compatible? Prove that these commute.

b) What is the matrix form of the spin operator S_z in a basis where its eigenvectors are given by $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$?

c) Imagine a system that would contain two such $s = 1$ particles. What values can the total spin of the system be (i.e. which spin ladders does this include)? Moreover, construct the states of the longest ladder in terms of the spin states of the two separate particles.